NEW METHOD FOR ANALIZING OF COMMON OPERATION OF CASCADE CONNECTED HYDRO POWER STATIONS

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ABSTRACT

The article shows a mathematical model for determining common operation of cascade-conected hydro power stations on the same water course or on a different ones. It will be defined: the objective function and all necessery constraints, which have to be satisfied, in form of equations or nonequations. The model is all in, becouse of including hidraulic losses in the income and outcome organs of the power stations, losses in turbines (which themselves depend of the turbine discharge and its available net head) and generators as wel as the functional dependence of the upper level to total water volum. At the same time condense solving of the problem will be shown as well as its usage on a concrete exemple. Aforementioned exemple is concerned to the existing cascade-conected hydro power stations HPP Kozjak, HPP Matka 2 and HPP Matka all on the river Treska.

INTRODUCTION

In the paper I will introduce formation of a mathematical model for determining the combined work of cascade – connected hydroelectric power generation plants which can be on the same or different water stream. The model takes into consideration the following aspects: loss in the inlets and outlets with the both hydroelectric power generation plants; coefficients dependence on the useful influence from the intake and the height appropriate for both hydroelectric power generation plants; the dependence of the upper quotas of the accumulation on their geometrical volume as well as the physical limits of the turbine units, the generators and the needs with reference to the biological minimum of the power plants which are hydraulically connected.

The efficiency of the model will be verified by performing simulation on the work of the HPP St. Petka and HPP Matka in hydrologic conditions which appeared in the period of 1946-1995. The simulation of the chain of work of the previously mentioned power plants will be performed on a typical work day in every month in a chronological chain i.e. for total 50x12=6000 hydrologic situations. In every hydrologic situation, the optimal usage of both power plants will be simulated and at the same time the optimal flow in the turbines of both power plants will be determined as well as the expected production of electric power of both power plants.

FORMATION OF A MATHEMATICAL MODEL

With a given regime of releasing the water flow in the hydroelectric power plant "Kozjak" within 24 hours with the suitable interflow $(q_t^{SP}, t = 1,24)$, the produced electric power on the behalf of the hydroelectric power plant "Sveta Petka" is:

$$E_{SP} = 9.81 \cdot \sum_{t=1}^{24} Q_t^{SP} \cdot H_{n,t}^{SP} \cdot \eta_t^{SP} (Q_t^{SP}, H_{n,t}^{SP}) \cdot \Delta t_t \qquad (1)$$

where:

$$Q_t^{SP} = q_t^{SP} - \omega(h_t^{SP}) \cdot \frac{dh_t^{SP}}{dt} \tag{2}$$

$$H_{n,t}^{SP} = h_t^{SP} - \alpha_1 \cdot \left[q_t^{SP} - \omega(h_t^{SP}) \cdot \frac{dh_t^{SP}}{dt} \right]^2 - \Delta h_{O,t}^{SP} \tag{3}$$

$$\begin{aligned} \eta_t^{SP}\left(Q_t^{SP}, H_{n,t}^{SP}\right) = & \left[a_0 + a_1 \cdot H_{n,t}^{SP} + a_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{M2}\right)^0 + \\ & + \left[b_0 + b_1 \cdot H_{n,t}^{SP} + b_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{SP}\right)^1 + \\ & + \left[c_0 + c_1 \cdot H_{n,t}^{SP} + c_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{SP}\right)^2 \end{aligned} \tag{4}$$

Index for the equations (1) to (4):

Q_t^{SP}	Total intake of water flow in the turbine of the HPP “Sveta Petka”, in time function;
$H_{n,t}^{SP}$	Net water head of the HPP “Sveta Petka” in time function;
η_t^{SP}	Coefficient of the beneficial use of the HPP “Sveta Petka” according to the existing project documentation in time function of intake, net water head and the optimal number of aggregates;
$\omega(h_t^{SP})$	Dependence of the mirror surface of HPP “Sveta Petka” on the quota of the accumulation in time function;
n_t^{SP}	optimal number of aggregates at that moment;
$\Delta h_{O,t}^{SP}$	Loss in the height of the outflow of the HPP “Sveta Petka”, in time function;
$a_i,\ i = 0,1,2$ $b_i,\ i = 0,1,2$ $c_i,\ i = 0,1,2$	Coefficients of the characteristics of the beneficial use;
α_2	Coefficient which defines the characteristics of the inlets of the HPP “Sveta Petka” on which the loss of intake is dependent. For HPP “Sveta Petka” it is $\alpha_2 = 0{,}000302$

In a similar way, the electric power produced on behalf of HPP “Matka”, is:

$$E_{MT} = 9.81 \cdot \sum_{t=1}^{24} Q_t^{MT} \cdot H_{n,t}^{MT} \cdot \eta_t^{MT}(Q_t^{MT}, H_{n,t}^{MT}) \cdot \Delta t_t \tag{5}$$

where:

$$Q_t^{MT} = q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \tag{6}$$

$$H_{n,t}^{MT} = h_t^{MT} - \alpha_2 \cdot \left[q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \right]^2 - \Delta h_{O,t}^{MT} \tag{7}$$

$$\begin{aligned} \eta_t^{MT}\left(Q_t^{MT}, H_{n,t}^{MT}\right) = & \left[d_0 + d_1 \cdot H_{n,t}^{MT} + d_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^0 + \\ & + \left[e_0 + e_1 \cdot H_{n,t}^{MT} + e_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^1 + \\ & + \left[f_0 + f_1 \cdot H_{n,t}^{MT} + f_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^2 \end{aligned} \tag{8}$$

It should be mentioned that the dimensions with SP refer to HPP “Sveta Petka”, and MT refer to HPP “Matka”. At the same time it should be mentioned that the outflow of HPP “Matka” as well as its net water head and the coefficient of beneficial use for different number of aggregates in the plant (at certain moment) depends on the outflow of HPP "Sveta Petka" at that same moment.

The index entries of equations (5) to (8) have analogue meaning to the entries in the equations (1) to (4) which were already described.

The hydraulic loss of the intake and outflow (given as height, in meters), of HPP "Matka", accordingly, is:

- in intake:

$$\Delta H_{dovod} = \alpha_1 \cdot \left[q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \right]^2 = 0.0004 \cdot \left[q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \right]^2 \quad (9)$$

- in outflow:

$$\Delta H_{odvod} = \beta \cdot \sqrt{q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt}} = 0.33541 \cdot \sqrt{q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt}} \quad (10)$$

where β is determined by the condition that the loss in the outflow is 1,5m with intake in the plant amounting to $20 m^3/s$, i.e.:

$$\beta = \frac{1.5}{\sqrt{20}} = 0.33541 \quad (11)$$

The quota of the entrance of the outflow of HPP "Matka" is 289,5 m.

In the equations (9) and (10) q_t^{MT} – represents the interflow between HPP "Sv.Petka" and HPP "Matka", and Q_t^{SP} – outflow in the turbine of HPP "Sv.Petka". The other index entries have the same meaning as the entries of HPP "Sv.Petka".

The total production of electric energy produced in both HPP "Matka" and HPP "Sv.Petka" is:

$$E_\Sigma = E_{SP} + E_{MT} \Rightarrow \max \quad (12)$$

It is essential that with the water flow in HPP "Sv.Petka" and the water flow in HPP "Matka" within 24 hours is produced maximum electric energy. For that purpose, the following constraints must be respected, given in equations:

$$\varphi_1 = \sum_{t=1}^{24} Q_t^{MT} \cdot \Delta t_t - V_{MT}^{24} = 0 \quad (13)$$

$$\varphi_2 = \sum_{t=1}^{24} Q_t^{SP} \cdot \Delta t_t - \left(V_{MT}^{24} + q_t^{MT,SP}\right) = 0 \quad (14)$$

In the equation (22) $q_t^{MT,SP}$ represents the inter water flow between the HPP "Matka" and HPP "Sv.Petka".

Finding the maximum of the function (8) with the constraints in (9) and (10) is equivalent to finding the extreme for the following aim function with no constraints:

$$\Phi = E_\Sigma + \lambda_1 \cdot \varphi_1 + \lambda_2 \cdot \varphi_2 \Rightarrow \max \quad (15)$$

where λ_1 and λ_2 are Lagrange multiplicators in consistence of the constraints in a form of equations which need to be determined.

Besides the constraints given in the equations (13) and (14), it is important that the following constraints are satisfied in nonequation form:

$$5{,}6 \text{ m}^3/\text{s} \le Q_t^{MT} \le 39{,}5 \text{ m}^3/\text{s} \qquad t = 1, \ldots., 24 \tag{16}$$

$$30{,}0 \text{ m}^3/\text{s} \le Q_t^{SP} \le 100 \text{ m}^3/\text{s} \qquad t = 1, \ldots.., 24 \tag{17}$$

At the same time the dimension 5,6 m³/s presents the biological maximum of HPP "Matka", and 39.5 m³/s and 100.0m³/s are, respectively, installed intake of HPP "Matka" and HPP "Sv.Petka".

Technical characteristics of HPP "Sv.Petka".

The hydroelectric power plant HPP "Sv.Petka" is designed for installed intake of 100m³/s and to work in a regime of extreme encumbrance. Its usable volume of accumulation, which is 1,110 m³/s is between the quotas 355m and 357,3m altitude above sea level. The dependence of the mirror surface of accumulation on the quota of the surface of accumulation is given in the following equation:

$$\omega(h_t^{SP}) = 0{,}248 + 0{,}0466 \cdot \left(h_t^{SP} - 350\right) - 0{,}00146 \cdot \left(h_t^{SP} - 350\right)^2 \cdot 10^6, \text{ m}^2 \tag{1}$$

where h_t represents the quota of accumulation of HPP "Sv.Petka".

In HPP "Sv.Petka" the installation of two hydro aggregates is planned, each for installed intake of 50 m³/s. The planned type of hydro turbines is Francise with the characteristics of beneficial use of the functions of height, water flow and the number of aggregates in a plant, is given in the following equation:

$$\begin{aligned} \eta_t^{SP}\left(Q_t^{SP}, H_{n,t}^{SP}\right) = & \left[a_0 + a_1 \cdot H_{n,t}^{SP} + a_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{SP}\right)^0 + \\ & + \left[b_0 + b_1 \cdot H_{n,t}^{SP} + b_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{SP}\right)^1 + \\ & + \left[c_0 + c_1 \cdot H_{n,t}^{SP} + c_2 \cdot \left(H_{n,t}^{SP}\right)^2\right] \cdot \left(Q_t^{SP} / n_t^{SP}\right)^2 \end{aligned} \tag{2}$$

At the same time:

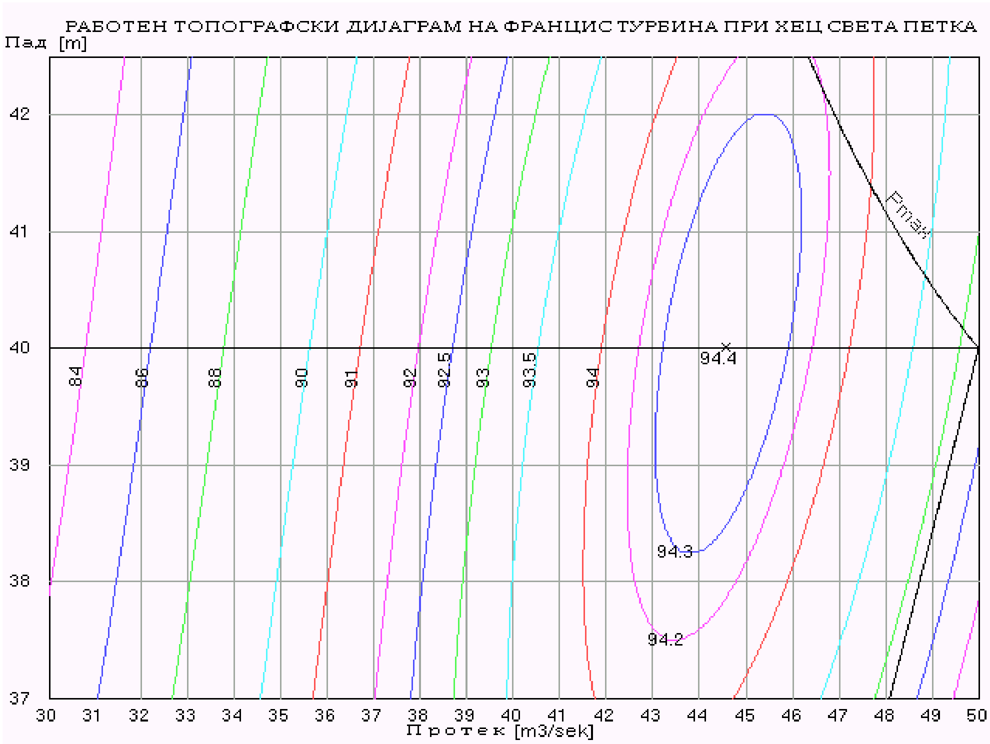
$$\begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ c_0 & c_1 & c_2 \end{bmatrix} = \begin{bmatrix} 350.5796000 & -17.6401300 & 0.2150553 \\ -9.1234730 & 0.72069880 & -0.0093673 \\ 0.0488688 & -0.00594989 & 0.0000851 \end{bmatrix} \tag{3}$$

Q_t^{SP} – intake of water in HEPP "Sv.Petka" in time interval *t*.

n_t^{SP} - number of aggregates in a plant in time interval *t*.

$H_{n,t}^{SP}$ – net water head of HEPP "Sv.Petka" in time interval *t*.

TOPOGRAPHICAL DIAGRAM OF FRANCISE TURBINE OF HEPP SVETA PETKA



Picture no.1. Characteristics of the hydro turbines in HEPP “Sv.Petka”:

$Q_i = 50$ m^3/s; $H_{\max} = 43{,}5$ m; $H_{\min} = 37{,}5$ m; $H_{konst.} = 40{,}0$ m

The hydraulic loss in the intake and outflow, given as height in meters, of HPP “Sv.Petka” in the inlets, is:

$$\Delta H_{dovod} = \alpha_2 \cdot \left[q_t^{SP} - \omega(h_t^{SP}) \cdot \frac{dh_t^{SP}}{dt} \right]^2 = 0.0004423 \cdot \left[q_t^{SP} - \omega(h_t^{SP}) \cdot \frac{dh_t^{SP}}{dt} \right]^2 \quad (4)$$

In the equation (4) q_t^{SP} – represents the intake in the accumulation of HPP “Sv.Petka” which actually is the total of the outflow of HPP “Kozjak” and the interflow between HEPP “Kozjak” and HPP “Sv.Petka”.

The quota of the entrance of the outflow of HPP “Sv.Petka” is 313.5m. The loss in its outflow depends on the level of the quota of accumulation “Matka” and is calculated with the relation:

$$\Delta H_{odvod} = A + B \cdot R_t^{MT} + C \cdot \left(R_t^{MT}\right)^2 \quad (5)$$

In the upper relation with R_t^{MT} refers to the higher level than the altitude of the lake Matka above the entrance of the water outflow of HPP “Sv.Petka” in the time interval *t*, i.e. the difference:

$$R_t^{MT} = K_{ota}^{MT} - 313.5 \quad (5a)$$

In cases when the level of Matka is under the quota 313.5m there would not be over levelling and the value of R_t^{MT} would be zero.

The value of the coefficients A, B and C depends on the intake and it is derived with the following relations:

$$\begin{aligned} A &= 0.033500 \cdot Q_t^{SP} - 0.0001300 \cdot \left(Q_t^{SP}\right)^2 \\ B &= -0.022653 \cdot Q_t^{SP} + 0.0001204 \cdot \left(Q_t^{SP}\right)^2 \\ C &= 0.003769 \cdot Q_t^{SP} - 0.0000225 \cdot \left(Q_t^{SP}\right)^2 \end{aligned} \quad (5b)$$

in which Q_t^{SP} refers to the intake in the turbines of HPP "Sv.Petka" in the time interval *t*.

From the upper relations the following loss of net head flow in the outflow of HEPP "Sv.Petka" is derived in metres:

Level of Matka	313.5	314.0	314.5	315.0	315.5	316.0	316.4
Q^{SP} = 50 m^3/sek	1.35	0.97	0.65	0.40	0.22	0.10	0.05
Q^{SP}= 100 m^3/sek	2.05	1.56	1.14	0.80	0.54	0.35	0.25

Technical characteristics of HPP "Matka"

The hydroelectric power plant HPP "Matka" is designed for installed intake of 39.5m³/s. Its usable volume of accumulation, which is 1,010 m³/s, is between the quotas 312m and 316,3m altitude above sea level.

The dependence of the mirror surface of accumulation on the quota of the surface of accumulation is given in the following equation:

$$\omega(h_t^{MT}) = 0{,}186 - 0{,}0097 \cdot \left(h_t^{MT} - 310\right) + 0{,}003413 \cdot \left(h_t^{MT} - 310\right)^2 \cdot 10^6,\ \mathrm{m}^2 \quad (6)$$

where h_t represents the quota of accumulation of HPP "Matka".

In HPP "Matka", besides the existing three aggregates with total installation 3x6,5m³/s = 19,5m³/s, installation of additional hydro aggregate is planned with installed intake of 20 m³/s, which will lead to installed intake of 39.5m³/s of HPP "Matka". In the analytical calculations the existing three aggregates will be treated as one equivalent aggregate with characteristics equal to the new aggregate which will be installed.

It is sustained that the type of the hydro turbine of the equivalent and the new aggregate will be Francise with the characteristics of beneficial use of the functions of height, water flow and the number of aggregates in a plant, given in the following equation:

$$\begin{aligned}\eta_t^{MT}\left(Q_t^{MT}, H_{n,t}^{MT}\right) = & \left[d_0 + d_1 \cdot H_{n,t}^{MT} + d_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^0 + \\ & + \left[e_0 + e_1 \cdot H_{n,t}^{MT} + e_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^1 + \\ & + \left[f_0 + f_1 \cdot H_{n,t}^{MT} + f_2 \cdot \left(H_{n,t}^{MT}\right)^2\right] \cdot \left(Q_t^{MT} / n_t^{MT}\right)^2 \end{aligned} \quad (7)$$

At the same time:

$$\begin{bmatrix} d_0 & d_1 & d_2 \\ e_0 & e_1 & e_2 \\ f_0 & f_1 & f_2 \end{bmatrix} = \begin{bmatrix} -159.87458215 & 18.33392865 & -.41489622 \\ 22.93614107 & -1.55085126 & .03549673 \\ -.58803946 & .03673624 & -.00082762 \end{bmatrix} \quad (8)$$

Q_t^{MT} – intake of water in HEPP "Matka" in time interval *t*.

n_t^{MT} - number of aggregates in a plant in time interval *t*.

$H_{n,t}^{MT}$ – net water head of HEPP "Matka" in time interval *t*.

Solving of the mathematical model

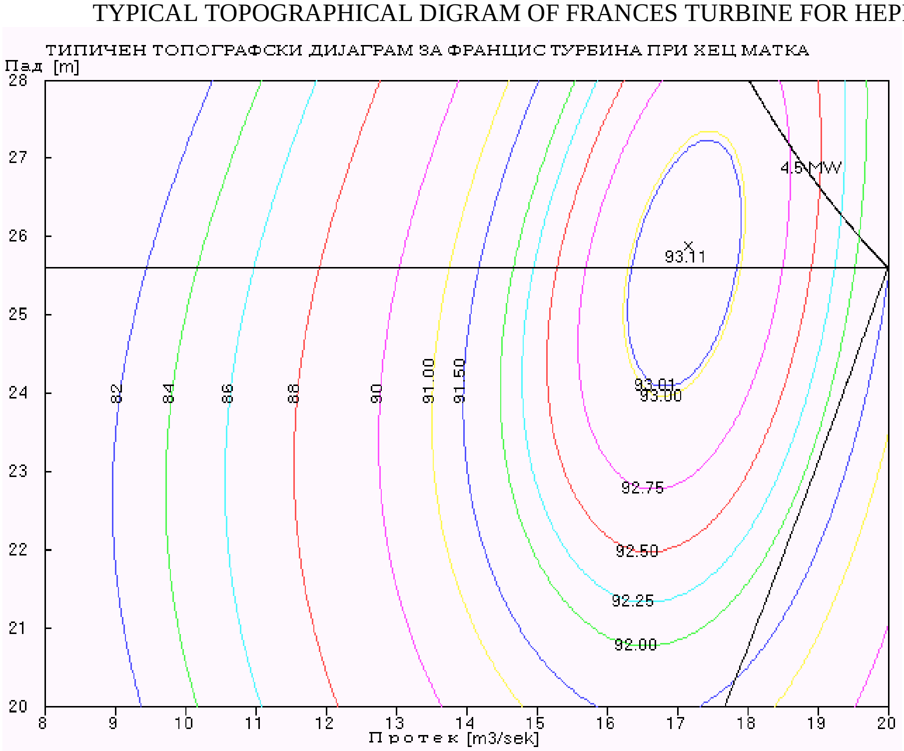
The formulation which defines the aim function is an exceptionally complex formulation, which makes it difficult to present the whole solving process within the frames of this paper. For its derivation it is necessary to form a Hessian matrix which determinants represent second partial derivatives of the aim function. It would cover over 50 pages to describe it which surpasses the limitations of this paper. Further more, it should be mentioned that after setting the conditions for the extreme and chain of transformations for derivation of the extreme function (20) with constraints given in equations (21) and (22) and constraints given in nonequations

(24) and (25) it amounts to solving a system of nonlinear equations (with high degree of nonlinearity) which consists of 50 equations for each hydrologic situation (the number of hydrologic situations is 600) with the same number of variables. The variable is the intake of HPP “Sv.Petka” and HPP “Matka” (24 each) as well as the Lagrange multiplicators λ_1 and λ_2. The number of such nonlinear systems is 50x12=600, and the total number of variables which needs to be determined is 600x50=30.000 variables.

Analysis of the results

Solving the model gives results for optimal production of electric energy form HPP “Matka2” and HPP “Matka” for all 600 analysed hydrologic situations. The frames of the study do not allow showing all of them as well as showing the optimal intake and optimal variations in the height for certain hydrologic situations. However, the results show that the intake is within its constraints and that the plants work in a way that they aspire to dispose the majority of their production in the period of high usage. In addition, the research shows that during the production of electric power the plants strive to work with maximum coefficient of beneficial usage.

TYPICAL TOPOGRAPHICAL DIGRAM OF FRANCES TURBINE FOR HEPP MATKA



Pic.2. Characteristics of hydro turbines in HPP “Matka”:

$Q_i = 20\ \text{m}^3/\text{s};\ H_{\max} = 28\ \text{m};\ H_{\min} = 20\ \text{m};\ H_{konst.} = 25.6\ \text{m}$

The hydraulic loss in the intake and the outflow (given in height, e.g. m), of HEPP “Matka”, accordingly, is:

- in the intake:

$$\Delta H_{dovod} = \alpha_1 \cdot \left[q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \right]^2 = 0.0004 \cdot \left[q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt} \right]^2 \quad (9)$$

- in the outflow:

$$\Delta H_{odvod} = \beta \cdot \sqrt{q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt}} = 0.33541 \cdot \sqrt{q_t^{MT} + Q_t^{SP} - \omega(h_t^{MT}) \cdot \frac{dh_t^{MT}}{dt}} \quad (10)$$

where β is derived by the condition that the loss in the intake is 1,5m in the downstream in the plant of 20 m^3/s, i.e.:

$$\beta = \frac{1.5}{\sqrt{20}} = 0.33541 \qquad (11)$$

The quota at the entrance of the outflow of HPP "Matka" is 289, 5 m.

In the equations (9) and (10) q_t^{MT} – represents interflow between HPP "Sv.Petka" and HPP "Matka", and Q_t^{SP} – the outflow in the turbines of HPP "Sv.Petka". The other entries have the same meaning as with HPP "Sv.Petka".

Conclusion

The paper presents a numerical method for determining the maximal production of electric energy from more hydroelectric power plants among which (or part of which) a hydraulic relation built on one or more water streams exists. The method also takes into consideration all of the relevant factors such as: loss in the intake and outflow in the power plants, loss in the turbines and generators, the type of turbines and their number and optimal changes in the quota of their accumulations. The method is applicable to phases of projecting new hydro-energy facilities (selection of dimensions of inlets, optimal usable volume of accumulation, optimal number and type of turbines) and to exploitation of the respective facilities (determining the favourable work regime).

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